

ALGEBRA REVIEW

Quadratic Formula

The roots of $ax^2 + bx + c = 0$

(if $a \neq 0$) are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example

The roots of $x^2 + 3x - 1 = 0$ are $x = \frac{-3 \pm \sqrt{13}}{2}$.

Exponents and Radicals

$$a^0 = 1, a \neq 0$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}} = (\sqrt[n]{a})^m$$

$$a^{-x} = \frac{1}{a^x}$$

$$(a^x)^y = a^{xy}$$

$$\sqrt{a} = a^{1/2}$$

$$\sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$$

$$a^x a^y = a^{x+y}$$

$$(ab)^x = a^x b^x$$

$$\sqrt[n]{a} = a^{1/n}$$

$$\sqrt[n]{\left(\frac{a}{b}\right)} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Special Factors

$$x^2 - a^2 = (x + a)(x - a)$$

$$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$$

$$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$$

Examples

$$(x^2 - 9) = (x + 3)(x - 3)$$

$$(x^3 - 8) = (x - 2)(x^2 + 2x + 4)$$

$$(x^3 - 4) = (x - \sqrt[3]{4})(x^2 + \sqrt[3]{4}x + \sqrt[3]{16})$$

Algebraic Errors to Avoid

$$\frac{a}{x+b} \neq \frac{a}{x} + \frac{a}{b}$$

To see this error, let $a = b = x = 1$.

$$\sqrt{x^2 + a^2} \neq x + a$$

$$a - b(x - 1) \neq a - bx - b$$

To see this error, let $x = 3$ and $a = 4$.

Remember to distribute the negative sign.

This should be:

$$a - b(x - 1) = a - bx + b$$

$$\frac{\left(\frac{x}{a}\right)}{b} \neq \frac{bx}{a}$$

To divide fractions, invert and multiply. This should be:

$$\frac{\left(\frac{x}{a}\right)}{b} = \frac{\left(\frac{x}{a}\right)}{b} \cdot \frac{1}{1} = \frac{x}{a} \cdot \frac{1}{b} = \frac{x}{ab}$$

$$\sqrt{-x^2 + a^2} \neq -\sqrt{x^2 - a^2}$$

We can't factor a negative sign out of the square root.

$$\frac{a + bx}{a} \neq 1 + bx$$

We can only cancel factors of the entire numerator with factors in the denominator.

This one should be:

$$\frac{a + bx}{a} = \frac{a}{a} + \frac{bx}{a} = 1 + \frac{bx}{a}$$

$$(x^2)^3 \neq x^5$$

The equation should be:

$$(x^2)^3 = x^2 x^2 x^2 = x^6$$