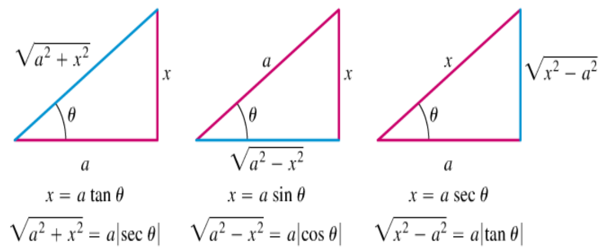
STUDENT SUCCESS CENTER

THE UNIVERSITY OF ALABAMA IN HUNTSVILLE

CALCULUS B REVIEW

Trig Sub Reference Triangle



Partial Fraction Decomposition

Given two functions, $P(x)$ and $Q(x)$, when integrating $\int \frac{P(x)}{Q(x)}dx$, and $P(x)$ is smaller in degree than $Q(x)$, the integral decomposes into:

$(ax+b)^k$ $\quad\quad$ $\frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \cdots + \frac{A_k}{(ax+b)^k}$

$(ax^2+bx+c)^k$ $\quad\quad$ $\frac{A_1x+B_1}{ax^2+bx+c} + \cdots + \frac{A_kx+B_k}{(ax^2+bx+c)^k}$

Trapezoidal Rule

To approximate $\int_a^b f(x)dx$, use

$$T = \frac{\Delta x}{2}(y_0 + 2y_1 + \cdots + 2y_{n-1} + y_n)$$

The y's are the values of f at the partition points

$$x_0 = a, x_1 = a + \Delta x, \ldots x_{n-1} = a + (n-1)\Delta x$$

$x_n = b$, Where $\Delta x = \frac{(b-a)}{n}$

Simpson's Rule

To approximate $\int_a^b f(x)dx$, use

$$S = \frac{\Delta x}{3}(y_0 + 4y_1 + 2y_2 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n$$

The number n is even, and $\Delta x = \frac{(b-a)}{n}$

Improper Integrals

Type I

1. If $f(x)$ is continuous on $[a, \infty)$, then

$$\int_a^\infty f(x)dx = \lim_{b\to\infty} \int_a^b f(x)dx$$

2. If $f(x)$ is continuous on $(-\infty, b]$, then

$$\int_{-\infty}^b f(x)dx = \lim_{a\to-\infty} \int_a^b f(x)dx$$

3. If $f(x)$ is continuous on $(-\infty, \infty)$, then

$$\int_{-\infty}^\infty f(x)dx = \int_{-\infty}^c f(x)dx + \int_c^\infty f(x)dx$$

Type II

1. If $f(x)$ is cont. on $(a, b]$ and discont. at a, then

$$\int_a^b f(x)dx = \lim_{c\to a^+} \int_c^b f(x)dx$$

2. If $f(x)$ is cont. on $[a, b)$ and discont. at b, then

$$\int_a^b f(x)dx = \lim_{c\to b^-} \int_a^c f(x)dx$$

3. If $f(x)$ is discont. at c, where $a < c < b$, and cont. on $[a, c) \cup (c, b]$, then

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

*In each case, if the limit is finite, the improper integrals **converges** and that the limit is the **value** of the improper integral. If the limit does not exist, the integral **diverges**.*

Polar Coordinates

Equations relating Polar and Cartesian

$$x = rcos\theta, \qquad y = rsin\theta,$$

$$tan\theta = \frac{y}{x}, \qquad r^2 = x^2 + y^2$$

Sequences

The sequence **converges** to the finite number L if the limit of the sequence

$$\lim_{n\to\infty} a_n = L$$

The sequence **diverges** to negative or positive infinity

$$\lim_{n\to\infty} a_n = \pm\infty$$

Infinite Series

Given a sequence of numbers $\{a_n\}$, $a_1 + a_2 + \cdots + a_n$, is an **infinite series**. The number a_n is the **nth term** of the series. The sequence $\sum_{k=1}^{n} a_k$ is the **sequence of partial sums** of the series. If the sequence of partial sums converges to a limit **L**, we say that the series **converges** and its sum is **L**. If the sequence of partial sums does not converge to a number, we say it **diverges**.

Convergence Tests

Comparison Test

Let $\sum a_n, \sum c_n, and\ \sum d_n$ be series with nonnegative terms. Suppose, for some number
N, that

$d_n \leq a_n \leq c_n$ For all $n > N$

(A) If $\sum c_n$ converges, then $\sum a_n$ also converges

(B) If $\sum d_n$ diverges, then $\sum a_n$ also diverges

P-Series Test

The *P*-Series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$, diverges if $p \leq 1$

Limit Comparison Test

Suppose that $a_n > 0$ and $b_n > 0$ for all $n \geq N$

If $\lim_{n\to\infty} \frac{a_n}{b_n}$ equals:

1) $c > 0$, then both series either converge or diverge

2) 0, and $\sum b_n$ converges, then $\sum a_n$ converges

3) ∞, and $\sum b_n$ diverges, then $\sum a_n$ diverges

Ratio Test

Let $\sum a_n$ be a series with positive terms, compute

$$\lim_{n\to\infty} \frac{|a_n+1|}{|a_n|} = p$$

Converges if $p < 1$, diverges if $p > 1$ or is infinite, inconclusive if $p = 1$

Root Test

Let $\sum a_n$ be a series with $a_n > 0$ for $n \geq N$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = p$$

Converges if $p < 1$, diverges if $p > 1$ or is infinite, inconclusive if $p = 1$

Alternating Series Test

$$\sum_{n=1}^{\infty} (-1)^{n+1} u_n = u_1 - u_2 + u_3 - u_4 \ldots$$

Converges if the following is satisfied: all u_n's are positive, the positive u_n's are non-increasing, and $u_n \to 0$

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