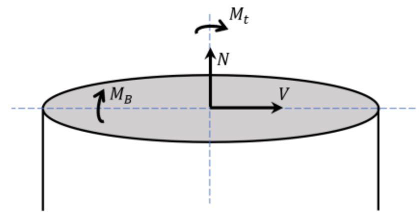
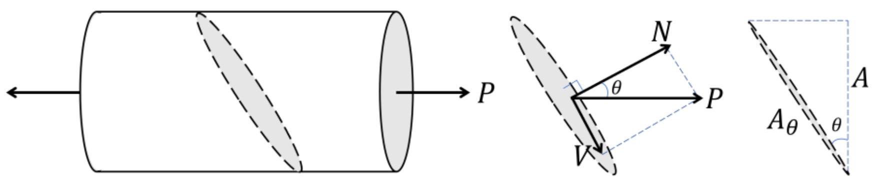
MECHANICS OF MATERIALS: STRESS & STRAIN

Stress:

- There are four types of forces:
    - Normal force (N) is perpendicular to the surface
    - Shear force (V) is parallel to the surface
    - Torsional moment (M_t) is about the axis normal (perpendicular) to the surface
    - Bending moment (M_B) is about the axis parallel to the surface



- Stress measures the intensity of the force per given area:
    - Normal stress (σ) results from the normal force N and/or bending moment M_B
    - Shear stress (τ) results from shear stress V and/or torsional moment M_t
- Stress can occur on oblique planes:



such that:

$N = Pcos\theta$

$V = Psin\theta$

$A_\theta = \dfrac{A}{cos\theta}$

$\sigma = \dfrac{N}{A_\theta} = \dfrac{Pcos^2\theta}{A}$

$\tau = \dfrac{V}{A_\theta} = \dfrac{Psin\theta cos\theta}{A}$

- Factor of safety is the ratio of failure load case to the necessary/typical load case:

$$FS = \frac{ultimate\ load}{allowable\ load} = \frac{ultimate\ stress}{allowable\ stress}$$

Strain:

- Strain is a measure of the material's response to stress and is expressed as a ratio to the change in length to the original length:

$$\varepsilon_{avg} = \frac{L_f - L_o}{L_o} = \frac{\Delta L}{L_o}$$

Where elongation results in a positive strain; compression results in a negative strain

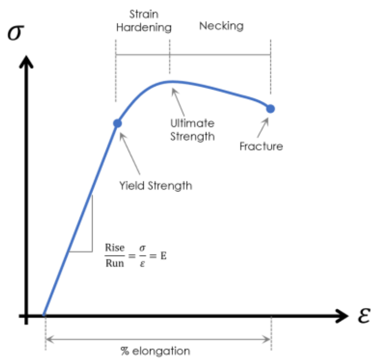
- Shear strain is based on the rotation of the object, measured in radians:

$$\gamma = \text{angle of deformation}$$

- Young's Modulus defines the relationship between normal stress and lateral strain:

$$E = \frac{\sigma}{\varepsilon}$$

This can be graphically represented on a stress-strain diagram (note it only holds for the elastic region) as the rise-run ratio. Young's Modulus is material-dependent and can be found in tables.



Elastic Region	Region in which an applied stress will cause a strain that is non-permanent. When the stress is removed, the material will return to its original state (reducing any elongation to 0).
Plastic Region	Region in which applied stresses cause irreversible strains. When the stress is removed, the material will return to the elongation of the yield point but will not reenter the elastic region.
Yield Strength	Dividing point between elastic behavior and plastic behavior *("point of no return")*. Any stress applied that is greater than the stress at the yield point results in bringing the material into the plastic region.
Brittle Material	A brittle material will experience miniscule elongation prior to failure.
Ductile Material	A ductile material will experience significant elongation *("necking")* before failure. Particularly, it may have a long period in the plastic region before the fracture point.
Ultimate Strength	The greatest stress that is experienced in a material. Ductile materials will experience less stress as necking increases. Brittle materials will have a fracture point very near (if not identical to) the ultimate strength.
Fracture Point	The point at which the material separates, or breaks. This point will be close to the ultimate stress for low ductility materials and further from the ultimate stress for high ductility materials.

- Modulus of Rigidity is similar to Young's Modulus but measures the ratio of shear stress to angle of deformation:

$$G = \frac{\tau}{\gamma}$$

- Poisson's Ratio measures the rate of lateral strain to axial strain; determining how likely a sample is to "neck" (think of taffy as it is pulled):

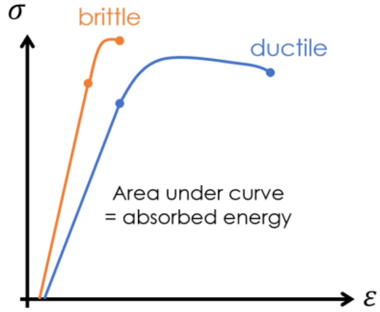
$$v = -\frac{lateral\ strain}{axial\ strain} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}\ ; \qquad \varepsilon_y = \varepsilon_z = -\frac{v\sigma_x}{E}\ ; \qquad -\varepsilon_{dia} = \frac{\Delta Dia}{Dia_o}$$

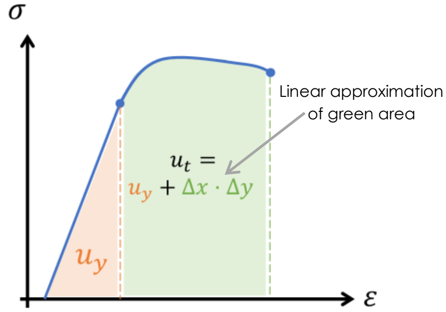
  A typical ratio is $0.2 - 0.4$. This indicates the "sideways" strain will be 40% of the "linear" strain.

- Relationship between these three (E, G, v) can be described using the following, such that knowing any two results in knowing the third:

$$E = 2G(1 + v)$$

- Strain energy observes how much energy is absorbed before entering the plastic deformation zone and in total before failure:





Because $work = \int_{x_0}^{x_f} F \cdot dx$, and Hooke's Law $F = kx$ ...

$$u = \int_0^{x_1} (kx)dx = \frac{1}{2}kx^2 = \frac{1}{2}Fx$$

$$\frac{u}{volume} = \int_0^{x_1} \frac{P}{A} \cdot \frac{dx}{L} = \int_0^{\varepsilon_1} \sigma_x d\varepsilon_x$$

$$u_y = \int_0^{\varepsilon_y} \sigma_x dx = E \int_0^{\varepsilon_y} \varepsilon_x d\varepsilon_x = \frac{E}{2}\varepsilon_y^2 = \frac{\sigma_y^2}{2E} = u_y$$

$$u_t = u_y + \Delta y \cdot \Delta x$$

For more information, visit a tutor. All appointments are available in-person at the Student Success Center, located in the Library, or online. Adapted from Hibbeler, R.C. (2014). *Mechanics of Materials* (9th Edition). Boston, MA: Prentice Hall.