STUDENT SUCCESS CENTER
THE UNIVERSITY OF ALABAMA IN HUNTSVILLE

MECHANICS OF MATERIALS: LOADINGS, TRANSFORMATIONS, & BEAM DESIGN

Combined Loadings:

- Thin-walled pressure vessels:
  1. Hoop stress is circumferential direction: $\sigma_1 = \frac{pr}{t}$
  2. Longitudinal stress is lengthwise: $\sigma_2 = \frac{pr}{2t} = \frac{1}{2}\sigma_1$

Where
r *= inner radius*
t *= wall thickness*

For spherical tanks,
$\sigma_1 = \sigma_2 = \frac{pr}{2t}$

- Combined loading due to normal force and bending:

$$\sigma_x = \frac{P}{A} + -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

- Combined loading due to shear forces:

$$\tau = \frac{T\rho}{J} + \frac{V_y Q_y}{It} + \frac{V_z Q_z}{It}$$

Stress Transformations:

- By rotating the coordinate axis we see all possible combinations of normal and shear stress. They are plotted as Mohr's Circle. This allows identification of coordinates in which all stress is normal and potentially points where all stress is shear.

$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cdot \cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cdot \cos(2\theta) - \tau_{xy}\sin(2\theta)$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \cdot \sin(2\theta) + \tau_{xy} \cdot \cos(2\theta)$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

such that $R^2 = \left(\sigma_{x'} - \sigma_{avg}\right)^2 + \tau_{xy}^2$

Circle of Radius R centered at σ_{avg} with no height

To rotate by θ_P into the principal stress plane (where $\tau_{xy} = 0$ and σ_x is max:

$$\tan(2\theta_P) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

To rotate by θ_S into the principal shear plane (where τ_{xy} is max and σ_x is min):

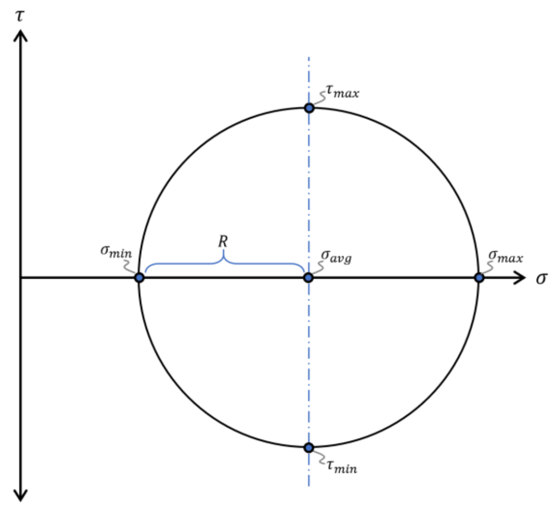
$$\tan(2\theta_s) = \frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

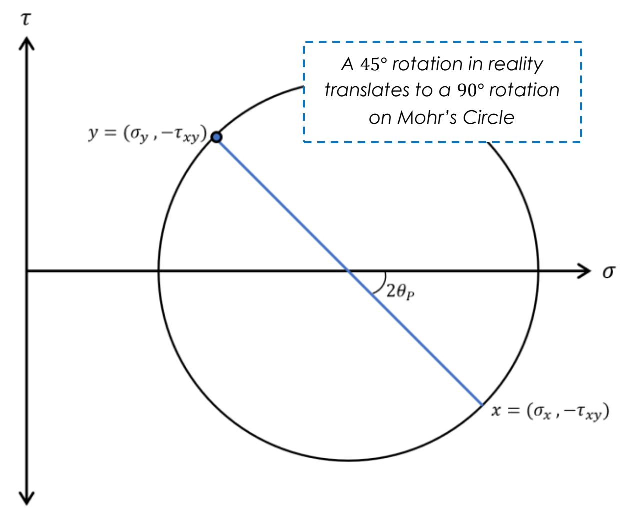
Negative reciprocal

σ is maximized and minimized at

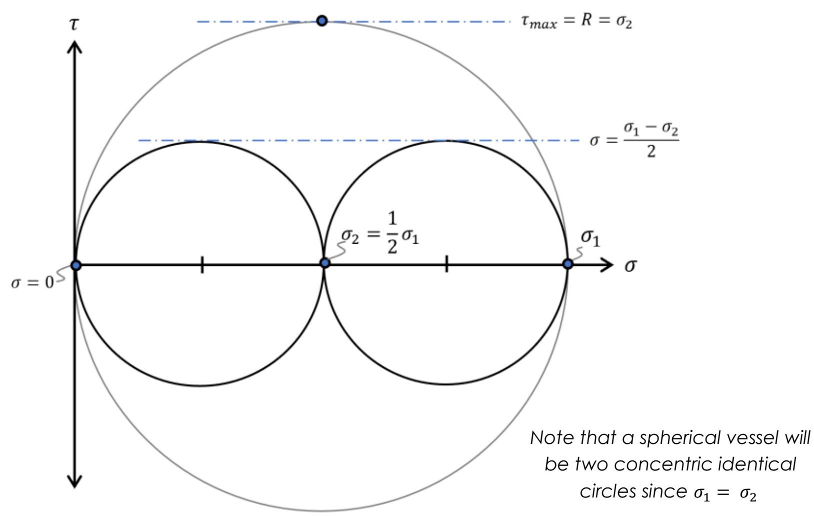
$$\underbrace{\frac{\sigma_x + \sigma_y}{2}}_{\sigma_{avg}} \pm \underbrace{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}_{R}$$

$\sigma_{avg} \pm R$





Pressure Vessels



Beam Design:

- Determine all beams with $S \geq S_{min}$ and select the one with the lowest $\frac{weight}{length}$ ratio (less weight is more desirable so long as it meets the minimum criterion set by S_{min}).
- $S = \frac{I}{c}$ and $\sigma = \frac{Mc}{I}$ therefore $S = \frac{I}{\frac{I\sigma}{M}} = \frac{M}{\sigma}$

$$S_{min} = \frac{|M_{max}|}{\sigma_{allow}}$$

Max moment is often found via shear and moment diagrams while allowable stress is usually supplied as a design criterion

For more information, visit a tutor. All appointments are available in-person at the Student Success Center, located in the Library, or online. Adapted from Hibbeler, R.C. (2014). *Mechanics of Materials* (9th Edition). Boston, MA: Prentice Hall.