CAL I: EXTREME VALUES

Relative Extreme Values:

First Derivative Test:

- Local maximum at x = c if f ‘(c) = 0 and the function changes from increasing to decreasing at x = c and f(c) is defined
- Local minimum at x = c if f ‘(c) = 0 and the function changes from decreasing to increasing at x = c and f(c) is defined

Second Derivative Test:

- Local maximum at x = c if f ‘(c) = 0 and f “(c) < 0 and f(c) is defined
- Local minimum at x = c if f ‘(c) = 0 and f “(c) > 0 and f(c) is defined

Example: Find relative max./min. of $f(x) = 3x^2 - 12x + 4$.

First Derivative Test:

- $f'(x) = 6x - 12 = 0$
  Critical Point at x=2
- On $(-\infty, 2)$, let the test point be x=0.
- $f'(0) = 6(0) - 12 =- 12 < 0$
  f(x) is decreasing on interval $(-\infty, 2)$
- On $(2, \infty)$, let the test point be x=3.
- $f'(3) = 6(3) - 12 = 18 - 12 = 6 > 0$
  f(x) is increasing on interval $(2, \infty)$
- Since f(x) changes from decreasing to increasing at the critical point x=2, x=2 is a relative minimum.
- Define f(c)

Second Derivative Test:

- $f'(x) = 6x - 12 = 0$
  Critical Point at x=2
- $f''(x) = 6$
- $f''(2) = 6 > 0$
  f(x) is concave up at x=2
- Since f(x) is concave up at the critical point x=2, x=2 is a relative minimum
  - Define f(c)

Absolute Extreme Values:

1. Find critical points
2. Evaluate f(x) for critical points and endpoints
3. Compare f(x) values to determine maximum and minimum

Example: Find absolute max. and min. of $f(x) = x^3 - 3x + 5$ on the interval [0,2].

- $f'(x) = 3x^2 - 3 = 0$
  Critical points are x=-1 and x=1

- $f(0) = (0)^3 - 3(0) + 5 = 5$

$$f(1) = (1)^3 - 3(1) + 5 = 3$$

$$f(2) = (2)^3 - 3(2) + 5 = 7$$

Critical point x=-1 is not included because -1 is not on the interval [0,2]

- Absolute maximum at x=2 and absolute minimum at x=1.