CAL I: LIMITS & INTEGRALS

Limits:

- Limit Properties
  - $\lim_{x\to a} [c\cdot f(x)] = c\cdot \lim_{x\to a} f(x)$
  - $\lim_{x\to a} [f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
  - $\lim_{x\to a} [f(x)\cdot g(x)] = \lim_{x\to a} f(x)\cdot \lim_{x\to a} g(x)$
  - $\lim_{x\to a} [\frac{f(x)}{g(x)}] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ $if \lim_{x\to a} g(x) \neq 0$
  - $\lim_{x\to a} [f(x)]^n = [\lim_{x\to a} f(x)]^n$
- Continuity Test – Used to determine if a function is continuous at a point.
  1. $f(c)\ exists$
  2. $\lim_{x\to c} f(x) exists,\ so \lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$
  3. $\lim_{x\to c} f(x) = f(c)$

  - Example: Determine if $f(x) = x^2 + 2x - 6$ is continuous at x=1.
    - $f(1) = (1)^2 + 2(1) - 6 =- 3$
    - $x^2 + 2x - 6 =- 3$
    - $x^2 + 2x - 6 =- 3$
    - The limit exists at x=1 because $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$.
    - The function is continuous at x=1 because $\lim_{x\to 1} f(x) = f(1)$.

Integrals:

- Definition – The sum of infinitely many, infinitesimally small areas.
  - $\int_a^b f(x)\, dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\, \Delta x$
  - $where\ \Delta x = \frac{b-a}{n}\ and\ x_k = a + k\cdot\Delta x$
- Integral Properties
  - $\int_a^a f(x)dx = 0$
  - $\int_b^a f(x)dx = -\int_a^b f(x)dx$
  - $\int_a^b c\cdot f(x)dx = c\cdot\int_a^b f(x)dx$
  - $\int_a^b [f(x) \pm g(x)]\, dx = \int_a^b f(x)\, dx \pm \int_a^b g(x)\, dx$

- $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

- Fundamental Theorem of Calculus
  - If f(x) is continuous on [a,b] and F '(x) = f (x):
    - $F'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$
    - $\int_a^b f(x)\,dx = F(b) - F(a)$
- Integration by Substitution
  - $\int_a^b [f(g(x))\cdot g'(x)]\,dx = \int_{g(a)}^{g(b)} f(u)\,du$
  - $where\ u = g(x)\ and\ du = g'(x)\,dx$
  - Example: Evaluate $\int_1^3 3(3x-1)^2\,dx$
    - Let $u = 3x - 1$, so $du = 3\,dx$.
    - $\int_{3(1)-1}^{3(3)-1} (u)^2\,du = \int_2^8 u^2\,du = \left[\frac{1}{3}u^3\right]_2^8$
    - $= \frac{1}{3}(8)^3 - \frac{1}{3}(2)^3 = 168$
    - So, $\int_1^3 3(3x-1)^2\,dx = 168$.
- Integration by Parts
  - $\int u\,dv = uv - \int v\,du$ where $v = \int dv$
    - OR
  - $\int f(x)\,g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$
  - Example: Evaluate $\int 3x^2 \bullet \ln ln\,x\ dx$
    - Let $u = \ln ln\,x$ and $dv = 3x^2 dx$.
    - So, $du = \frac{1}{x}dx$ and $v = \int 3x^2 dx = x^3$.
    - $\int 3x^2 \bullet \ln ln\,x\ dx = \ln ln\,x \bullet x^3 - \int x^3 \bullet \frac{1}{x}dx$
    - $= x^3 \ln ln\,x - \int x^2 dx = x^3 \ln ln\,x - \frac{1}{3}x^3 + C$

- Common Integrals
  - $\int k\, dx = k \cdot x + C$
  - $\int e^{x}\, dx = e^{x} + C$
  - $\int \sin sin\,(x)\, dx = -\, cos(x) + C$
  - $\int sec^{2}(x)dx = \tan tan\,(x) + C$
  - $\int \sec sec\,(x) \cdot \tan tan\,(x)\, dx = \sec sec\,(x) + C$
  - $\int x^{n}\, dx = \frac{1}{n+1} x^{n+1} + C$
  - $\int \frac{1}{x}\, dx = ln|x| + C$
  - $\int \cos cos\,(x)\, dx = \sin sin\,(x) + C$
  - $\int csc^{2}(x)dx = -\cot cot\,(x) + C$
  - $\int csc(x) \cdot cot\,(x)\, dx = -\, csc(x) + C$

For more information, visit a tutor. All appointments are available in-person at the Student Success Center, located in the Library, or online. Adapted from Hass, J., Weir, M.D., & Thomas, G.B. (2012). *University Calculus: Early Transcendentals* (2nd ed.). Boston: Pearson Education.