CAL III: PARAMETERIZATION, TNB FRAME OF REFERENCE, & EXTREME VALUES

Parameterization:

- Curves in space can be parameterized using intermediate variables, such as t (which can be thought of as *time*). When $r(t) = f(t)i + g(t)j + h(t)k$, the head of the vector traces out the path of the curve as a function of t over some (time) interval *I*. All differential rules hold true when a curve is parameterized.
- Introducing r as position and t as time allows for expansion to use $v = \frac{dr}{dt}$ for velocity and $= \frac{d}{dt}\left(\frac{dr}{dt}\right) = \frac{dv}{dt}$. This allows for simplification of formulas such as arc length:

$$L = \int_{t=a}^{t=b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt = \int_{t=a}^{t=b} \sqrt{|v|} dt$$

TNB Frame of Reference:

- Rather than defining the reference frame from a stationary observer of an airplane, it can be defined from an observer inside the airplane. This frame of reference is completed with 3 descriptions of motion:
  - The unit tangent vector, $\boldsymbol{T}$. This defines forward motion, normalized to be a unit vector:

$$T = \frac{\frac{dr}{ds}}{\left|\frac{dr}{ds}\right|} = \frac{v}{|v|}$$

  - The principal unit normal vector, $\boldsymbol{N}$. This defines the direction which the curve is turning and is, by definition, orthogonal to T. It is normalized by dividing the orthogonal vector by length of κ, which defines the intensity/amount of the turn.

$$N = \frac{1}{\kappa} \cdot \frac{dT}{ds} \quad \text{where} \quad \kappa = \frac{1}{|v|} \cdot \left|\frac{dT}{dt}\right| \quad \text{and therefore} \quad N = \frac{\frac{dT}{ds}}{\left|\frac{dT}{ds}\right|}$$

  - The binormal vector, $\boldsymbol{B}$. This is defined as the cross product of T and N and is used in finding the torsion, τ, which describes how the path twists.

$$\tau = -\frac{dB}{ds} \cdot N \quad \text{where } B = T \times N$$

- These are used in finding the acceleration vector, a:

Based on Pythagorean's Theorem

$$a = a_T \cdot T + a_N \cdot N \quad \text{where} \quad a_T = \frac{dv}{dt} = \frac{d^2s}{dt^2} \quad \text{and} \quad a_N = \kappa \cdot |v|^2 = \sqrt{|a|^2 - a_T^2}$$

For more information, visit a tutor. All appointments are available in-person at the Student Success Center, located in the Library, or online. Adapted from Hass, J., Weir, M.D., & Thomas, G.B. (2012). *University Calculus: Early Transcendentals* (2nd ed.). Boston: Pearson Education.