STUDENT SUCCESS CENTER
THE UNIVERSITY OF ALABAMA IN HUNTSVILLE

Calculus II - Integration Methods

Integration By Parts

$\int udv = uv - \int vdu$ where $v = \int dv$ OR$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$

- Example: $\int xcos(x)dx \rightarrow u = x,\ du = 1dx,\ dv = cos(x)dx,\ v = sin(x) \rightarrow$

$xsin(x) - \int sin(x)dx = xsin(x) + cos(x) + c$

Trig Integrals

Let's assume an integral of the form $\int sin^m x cos^n x dx$ such that m and n are nonnegative integers (positive or zero). We will divide the appropriate substitution into three cases according to m and n being odd or even.

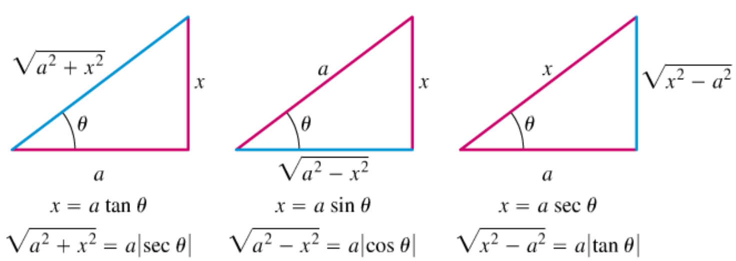
Case 1: If m is odd, let $m = 2k + 1$ and use the identity $sin^2 x = 1 - cos^2 x$ to get $sin^m x = sin^{2k+1} x = (sin^2 x)^k sinx = (1 - cos^2 x)^k sin(x)$ and do u-sub where u=cos(x)

Case 2: If m is even and n is odd, let $n = 2k + 1$ and use the identity $cos^2 x = 1 - sin^2 x$ to get $cos^n x = cos^{2k+1} x = (cos^2 x)^k cosx = (1 - sin^2 x)^k cos(x)$ and do u-sub where u=sin(x)

Case 3: If m is even and n is even, substitute $sin^2(x) = \frac{1-cos(2x)}{2}$, $cos^2(x) = \frac{1+cos(2x)}{2}$

For Integrals of Powers of tan(x) and sec(x): To integrate higher powers, use the identities $tan^2(x) = sec^2(x) - 1$ and $sec^2(x) = tan^2(x) + 1$ and integration by parts when necessary.

Trig Substitution



Case 1: If the integral contains $\sqrt{a^2 + x^2}$, then $x = atan\theta,\ dx = asec^2(\theta)d\theta$

Case 2: If the integral contains $\sqrt{a^2 - x^2}$, then $x = asin\theta,\ dx = acos(\theta)d\theta$

Case 3: If the integral contains $\sqrt{x^2 - a^2}$, then $x = a\sec\theta,\ dx = a\sec(\theta)\tan(\theta)d\theta$

Partial Fraction Decomposition

We assume a rational function $\frac{f(x)}{g(x)}$ such that the degree of $f(x)$ is less that the degree of $g(x)$ and we know the factors of $g(x)$.

1. Let $x - r$ be a linear factor of $g(x)$ and let $(x - r)^m$ be the highest power of $x - r$ that divides $g(x)$. Then we rewrite as $\frac{A_1}{(x-r)} + \frac{A_2}{(x-r)^2} + ... + \frac{A_m}{(x-r)^m}$ and do this for each distinct factor of $g(x)$.
2. Let $x^2 + px + q$ be a non-reducible quadratic factor of $g(x)$ where it has no real roots and let $(x^2 + px + q)^n$ be the highest power of $x^2 + px + q$ that divides $g(x)$. Then we rewrite as $\frac{B_1x+C_1}{(x^2+px+q)} + \frac{B_2x+C_2}{(x^2+px+q)^2} + ... + \frac{B_nx+C_n}{(x^2+px+q)^n}$ and do this for each distinct quadratic factor of $g(x)$.
3. Set the function $\frac{f(x)}{g(x)}$ equal to the sum of all of these partial fractions and clear the equation of fractions, arranging the terms in decreasing powers of x.
4. Equate the coefficients of corresponding powers of x and solve.

Improper Integrals

Type I (Infinite Bounds):

If $f(x)$ is continuous on $[a,\ \infty)$, then $\int_a^\infty f(x)dx = \lim_{b\to\infty} \int_a^b f(x)dx$

If $f(x)$ is continuous on $(-\ \infty,\ b]$, then $\int_{-\infty}^b f(x)dx = \lim_{a\to-\infty} \int_a^b f(x)dx$

If $f(x)$ is continuous on $(-\ \infty, \infty)$, then $\int_{-\infty}^\infty f(x)dx = \lim_{a\to-\infty} \int_a^c f(x)dx + \lim_{b\to\infty} \int_c^b f(x)dx$ for some real number c.

Type II (Discontinuity):

If $f(x)$ is cont. on $(a, b]$ and discontinuous at a, then $\int_a^b f(x)dx = \lim_{c\to a^+} \int_c^b f(x)dx$

If $f(x)$ is cont. on $[a, b)$ and discontinuous at b, then $\int_a^b f(x)dx = \lim_{c\to b^-} \int_a^c f(x)dx$

If $f(x)$ is discontinuous at c, where $a < c < b$, and cont. on $[a, c)\cup(c, b]$, then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

In each case, if the limit is finite, the improper integral **converges and that the limit is the **value** of the improper integral. If the limit does not exist, the integral **diverges**.**

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