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Calculus II - Vectors

Vector Notation for 3-Dimensional Vector

- Component Form: $\vec{v} = \ < v_1, v_2, v_3 >$
- "i-j-k" Notation: $\vec{v} = v_i \hat{i} + v_2 \hat{j} + v_3 \hat{k}$

Magnitude or Length of a Vector:

$$\left|\vec{v}\right| = \sqrt{v_1^{\ 2} + v_2^{\ 2} + v_3^{\ 2}} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Vector Operations: Let $\vec{u} = \ < u_1, u_2, u_3 >$ and $\vec{v} = \ < v_1, v_2, v_3 >$ with c being a scalar. Then vector addition is defined by $\vec{u} + \vec{v} = \ < u_1 + v_1,\ u_2 + v_2,\ u_3 + v_3 >$ and scalar multiplication is defined by $c\vec{u} = \ < cu_1, cu_2, cu_3 >$

Unit Vector: Given that $\hat{v}$ is a nonzero vector, $\frac{\vec{v}}{\left|\vec{v}\right|}$ is the unit vector, also called the direction, of $\vec{v}$. The equation $\vec{v} = \left|\vec{v}\right| \frac{\vec{v}}{\left|\vec{v}\right|}$ expresses v as its length times its direction.

Dot Product of Two Vectors: $\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$

Properties of Dot Product:

1) $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
2) $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$
3) $0 \cdot \vec{v} = 0$
4) $(c\vec{u}) \cdot \vec{v} = \vec{u} \cdot (c\vec{v}) = c(\vec{u} \cdot \vec{v})$
5) $\vec{u} \cdot \vec{u} = |\vec{u}|^2$
6) Two vectors are orthogonal if $\vec{u} \cdot \vec{v} = 0$

Angle Between Two Nonzero Vectors: $\theta = cos^{-1}(\frac{\hat{u} \cdot \hat{v}}{|\hat{u}||\hat{v}|})$

Projection of $\vec{u}$ onto $\vec{v}$: $proj_{\hat{v}} \hat{u} = (\frac{\hat{u} \cdot \hat{v}}{|\hat{v}|^2})\ \hat{v}$

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Cross Product of Two Vectors: $\vec{u} \times \vec{v} = (|\hat{u}||\hat{v}|sin\theta)\ \hat{n}$ such that n is a unit vector perpendicular to the plane determined by $\vec{u}$ and $\vec{v}$.

Determinant Formula for Cross Product of Two Vectors: Suppose $\hat{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$ and $\hat{v} = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$. Then $\hat{u} \times \hat{v} = (u_2v_3 - u_3v_2)\ \hat{i} - (u_1v_3 - u_3v_1)\ \hat{j} + (u_1v_2 - u_2v_1)\ \hat{k}$.

Properties of Cross Product:

1) $(r\vec{u}) \times (s\vec{v}) = (rs)(\vec{u} \times \vec{v})$
2) $\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$
3) $\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$
4) $(\vec{v} + \vec{w}) \times \vec{u} = \vec{v} \times \vec{u} + \vec{w} \times \vec{u}$
5) $0 \times \vec{v} = 0$
6) $\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\ \vec{v} - (\vec{u} \cdot \vec{v})\ \vec{w}$
7) Two vectors are parallel if $\vec{u} \times \vec{v} = 0$