STUDENT SUCCESS CENTER
THE UNIVERSITY OF ALABAMA IN HUNTSVILLE

Linear Algebra Review

Matrix Properties

Matrix Addition: If $A = [a_{ij}]$ and $B = [b_{ij}]$ are matrices of size $m \times n$, then their sum is the $m \times n$ matrix given by $A + B = [a_{ij} + b_{ij}]$. The sum of two different size matrices is undefined.

Matrix Scalar Multiplication: If $A = [a_{ij}]$ is an $m \times n$ matrix and c is a scalar, then the multiple scalar of A by c is the $m \times n$ matrix given by $cA = [ca_{ij}]$.

Matrix Multiplication: If $A = [a_{ij}]$ is an $m \times n$ and $B = [b_{ij}]$ is an $n \times p$ matrix, then the product AB is an $m \times p$ matrix $AB = [c_{ij}]$ where $c_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj}$.

Properties of Matrix Addition and Scalar Multiplication:

1) $A + B = B + A$
2) $A + (B + C) = (A + B) + C$
3) $(cd)A = c(dA)$
4) $1A = A$
5) $c(A + B) = cA + cB$
6) $(c + d)A = cA + dA$

Properties of Matrix Multiplication:

1) $A(BC) = (AB)C$
2) $A(B + C) = AB + AC$
3) $(A + B)C = AC + BC$
4) $c(AB) = (cA)B = A(cB)$

Properties of Transposes:

1) $(A^T)^T = A$
2) $(A + B)^T = A^T + B^T$
3) $(cA)^T = c(A^T)$
4) $(AB)^T = B^T A^T$

Inverse of a Matrix: An $n \times n$ matrix A is invertible (or nonsingular) when there exists an $n \times n$ matrix B such that $AB = BA = I_n$ where I_n is the identity matrix. B is called the inverse of A. A matrix that does not have an inverse is noninvertible (or singular).

Properties of Inverse Matrices: If A is an invertible matrix, k is a positive integer, and c is a nonzero scalar, then $A^{-1}, A^k, cA, and\ A^T$ are invertible and the following are true:

1) $(A^{-1})^{-1} = A$
2) $(A^k)^{-1} = (A^{-1})^k$
3) $(cA)^{-1} = \frac{1}{c}A^{-1}$

4) $(A^T)^{-1} = (A^{-1})^T$

Inverse of a Product: If A and B are invertible matrices of order n, then AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$.

Systems of Equations with Unique Solutions: If A is an invertible matrix, then the system of linear equations $Ax = b$ has a unique solution given by $x = A^{-1}b$.

Properties of Determinants

Determinant of a Triangular Matrix: If A is a triangular matrix of order n, then its determinant is the product of the entries on the main diagonal. That is, $det(A) = |A| = a_{11}a_{22}... a_{nn}$.

Conditions That Yield a Zero Determinant: If A is a square matrix and any one of the following conditions are true, then $det(A) = 0$.

1) An entire row or column consists of zeros.
2) Two rows or columns are equal.
3) One row or column is a multiple of another row or column.

Determinant of Matrix Multiplication: If A and B are square matrices of order n, then $det(AB) = det(A)det(B)$.

Determinant of a Scalar Multiple of a Matrix: If A is a square matrix of order n and c is a scalar, then the determinant of cA is $det(cA) = c^n det(A)$.

Determinant of an Invertible Matrix: A square matrix is invertible if and only if $det(A) \neq 0$.

Determinant of an Inverse Matrix: If A is an $n \times n$ invertible matrix, then $det(A^{-1}) = \frac{1}{det(A)}$.

Equivalent Conditions for a Nonsingular Matrix: If A is an $n \times n$ matrix, then the following statements are equivalent:

1) A is invertible.
2) $Ax = b$ has a unique solution for every $n \times 1$ column matrix b.
3) $Ax = O$ has only the trivial solution.
4) A is row-equivalent to I_n.
5) A can be written as the product of elementary matrices.
6) $det(A) \neq 0$.

Inverse of a Matrix Given Its Adjoint: If A is an $n \times n$ matrix, then $A^{-1} = \frac{1}{det(A)} adj(A)$

Cramer's Rule: If a system of n linear equations in n variables has a coefficient matrix A with a nonzero determinant, then the solution of the system is

$x_1 = \frac{det(A_1)}{det(A)}$, $x_2 = \frac{det(A_2)}{det(A)}$, ... , $x_n = \frac{det(A_n)}{det(A)}$ where the ith column of A is the column of constants in the system of equations.