STUDENT SUCCESS CENTER
THE UNIVERSITY OF ALABAMA IN HUNTSVILLE

Loadings, Transformation, and Beam Design

Combined Loadings:

- Thin-walled pressure vessels:
  1. Hoop stress is circumferential direction: $\sigma_1 = \frac{pr}{t}$
  2. Longitudinal stress is lengthwise: $\sigma_2 = \frac{pr}{2t} = \frac{1}{2}\sigma_1$

     Where r= inner radius, t = wall thickness, and for spherical tanks, $\sigma_1 = \sigma_2 = \frac{pr}{2t}$

- Combined loading due to normal force and bending:

$$\sigma_x = \frac{P}{A} +- \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

- Combined loading due to shear forces:

$$\tau = \frac{T\rho}{J} + \frac{V_y Q_y}{It} + \frac{V_z Q_z}{It}$$

Stress Transformations:

- By rotating the coordinate axis we see all possible combinations of normal and shear stress. They are plotted as Mohr's Circle. This allows identification of coordinates in which all stress is normal and potentially points where all stress is shear.

$$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cdot cos(2\theta) + \tau_{xy} sin(2\theta)$$

$$\sigma'_y = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cdot cos(2\theta) - \tau_{xy} sin(2\theta)$$

$$\tau_{x'y'} = - \frac{\sigma_x - \sigma_y}{2} \cdot sin(2\theta) + \tau_{xy} cos(2\theta)$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + (\tau_{xy})^2} \text{ such that } R^2 = (\sigma'_x - \sigma_{avg})^2 + (\tau_{xy})^2$$

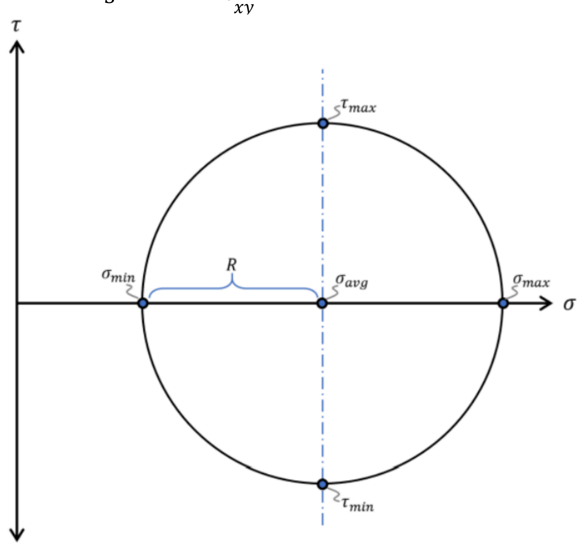
σ is maximized and minimized at $\frac{\sigma_x+\sigma_y}{2} \pm \sqrt{(\frac{\sigma_x-\sigma_y}{2})^2 + (\tau_{xy})^2} = \sigma_{avg} \pm R$

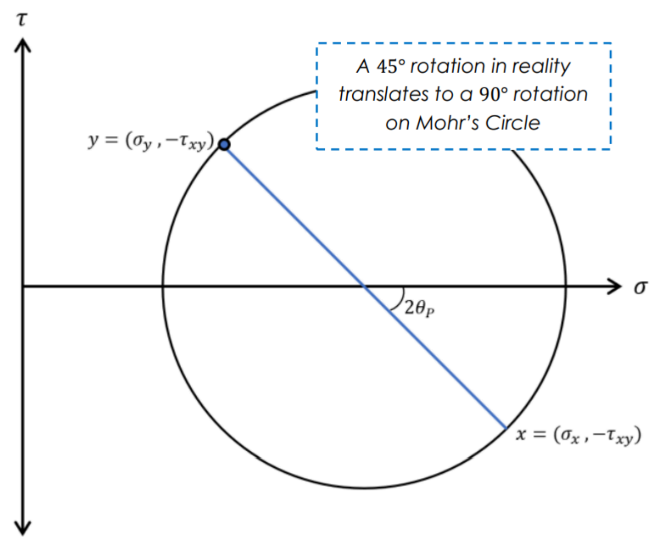
To rotate by θ_p into the principal stress plane (where $\tau_{xy} = 0$ and σ_x is max):

$$tan(2\theta_p) = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}$$

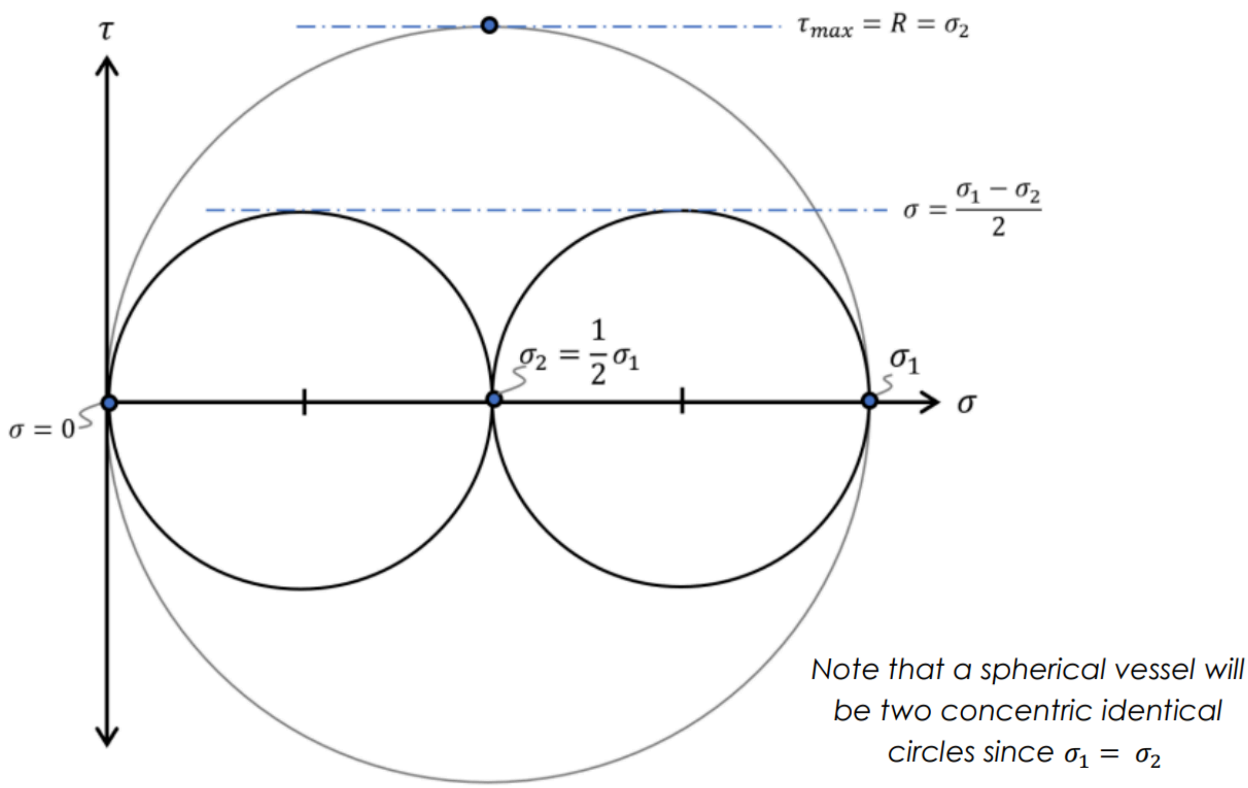
To rotate by θ_p into the principal shear plane (where τ_{xy} is max and σ_x is min):

$$tan(2\theta_s) = \frac{\sigma_x-\sigma_y}{2\tau_{xy}}$$





Pressure Vessels



Beam Design:

- Determine all beams with $S \geq S_{min}$ and select the one with the lowest $\frac{weight}{length}$ ratio (less weight is more desirable so long as it meets the minimum criterion set by S_{min}).

$$S = \frac{I}{c} \text{ and } \sigma = \frac{Mc}{I}, \text{ thus } S = \frac{M}{\sigma}$$

$$S_{min} = \frac{|M_{max}|}{\sigma_{allow}}$$

Max moment is often found via shear and moment diagrams while allowable stress is usually supplied as a design criterion