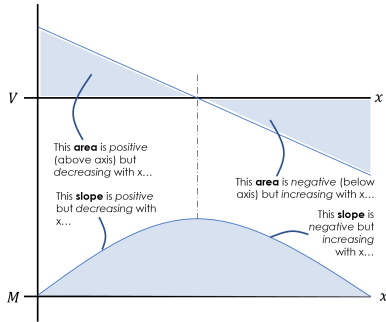


## MECHANICS OF MATERIALS: NORMAL & SHEAR STRESS

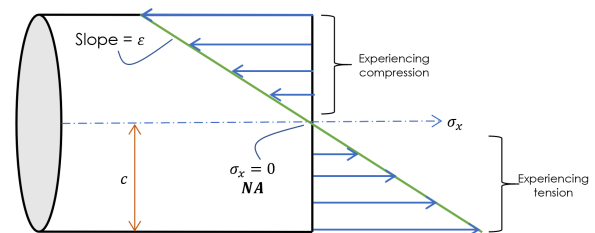
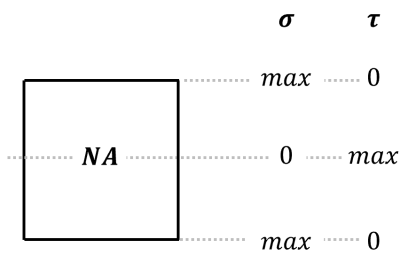
### Normal Stress Caused by Bending:

- Recall shear and moment calculation and graphing techniques:



Load ( $w$ )	Shear ( $V$ )	Moment ( $M$ )
0	0	0
$-w$ (constant)	$wx$ (linear)	$wx^2$ (parabolic)
$-wx$ (linear)	$wx^2$ (parabolic)	$wx^3$ (cubic)
$-wx^2$ (parabolic)	$wx^3$ (cubic)	$wx^4$ (quartic)

- The **Neutral Axis** is the axis at which a member in bending



experiences no normal stress:

- When bending, we observe how stress varies as a point moves away from the Neutral Axis.

$$\epsilon = -\frac{y}{c} \cdot \epsilon_{max} \quad \text{such that} \quad \sigma_x = E\left(-\frac{y}{c}\right) \cdot \epsilon_{max} \quad \text{and} \quad \sigma_x = -\frac{y}{c} \cdot \sigma_{max}$$

$$I = \int y^2 dA = \frac{Mc}{\sigma_{max}} \quad \text{or} \quad \sigma_x = -\frac{My}{I} \quad \text{for any } y$$

The negative sign is necessary because for a positive point, the beam experiences compression

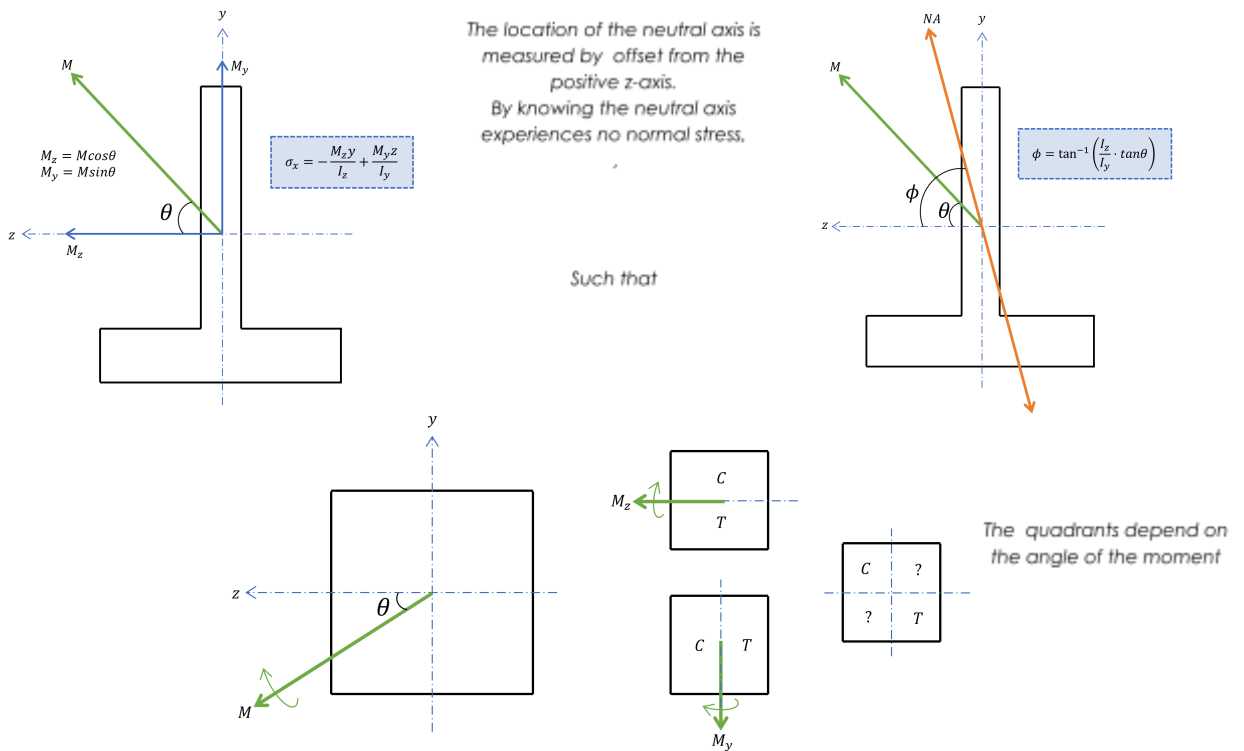
Rectangle	$I_{rectangle} = \frac{1}{12}bh^3$
Circle	$I_{circle} = \frac{1}{4}\pi R^4$
Triangle	$I_{triangle} = \frac{1}{36}bh^3$

- The **Parallel Axis Theorem** is often used for objects that are not strictly rectangular or circular, but rather are comprised of several shapes:

Semi-circle	$I_{\text{semicircle}} = \frac{1}{8} \pi R^4$
$b$ is in the direction parallel ( $\parallel$ ) to the NA $h$ is in the direction perpendicular ( $\perp$ ) to the NA	

$$I_i = I'_i + A_i d_i^2$$

- Unsymmetrical bending** due to moments at angles requires the moment vector be decomposed:

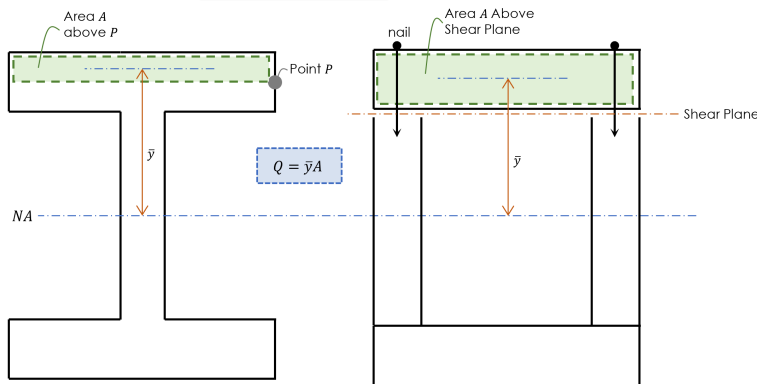


### Shear Stress Caused By Shear Force:

$\bar{y}$	Distance from NA to the centroid of A
A	Area above the point or above Shear Plane (opposite side of NA)

- There are two directions in which shear force acts: longitudinal (along the length of the beam) and transverse (on cut plane)
  1. Horizontal shear (longitudinal):  $\Delta H = \frac{VQ}{I} \Delta x$
  2. Transverse shear:  $\tau_{avg} = \frac{VQ}{It}$  and  $\tau_{max} = 1.5 \cdot \frac{V_{max}}{A}$
- Shear flow (shear force per unit length):  $q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I}$

**How to determine Q:**



$I$	Moment of inertia of entire object (independent of $A$ or $\bar{y}$ )
$V$	Transverse force applied
$t$	Thickness of the object at the point observed or Shear Plane