34th Annual University of Alabama System Applied Mathematics Meeting

University of Alabama in Huntsville

November 5, 2022

SCHEDULE

All talks will be in Room 107, Shelby Center for Science and Technology (SST), while the refreshments will be served in Room 107B, SST. We will have faculty and student discussion in Room 107 and Room 105, SST, respectively. We contacted our Parking Services, and were told that parking permits are not required on Saturday, and you can park wherever space is available on campus.

10:00 Refreshments in Room 107B, SST

10:15-10:25 Welcome remarks by the Department Chair **Dr. Toka Diagana**

10:25-11:05 **Dr. Brendan Ames** (UA) *When Can Machines Learn? Insights from Convex Optimization and Semidefinite Programming*

11:05-11:25 **Steven Redolfi** (UAB) *Fourier Transforms for ODEs Without the Unique Continuation Property*

11:25-11:45 **Quinten McKinney** (UAH) *The Stochastic Sigmoid Beverton-Holt Equation*

11:45-2:00 Lunch at area restaurants

2:00-2:40 **Dr. Toka Diagana** (UAH) *Well-posedness and Stability Results for Some Nonautonomous Second-Order Evolution Equations*

2:40-3:00 **Sylvia Amihere** (UA) *Benchmarking Electrostatic Free Energy of the Nonlinear Poisson-Boltzmann Model for the Kirkwood Sphere*

3:00-3:40 **Dr. Keren Li** (UAB) *Response-Aided Score-Matching Approaches for Distributed Learning under Generalized Linear Models*

3:40 Refreshments in Room 107B, SST

4:00-4:30 Faculty Discussion in Room 107, SST

4:00-4:30 Student Discussion in Room 105, SST

ABSTRACTS

When Can Machines Learn? Insights from Convex Optimization and Semidefinite Programming *Brendan Ames* (UA)

Abstract: Recent years have seen an incredible increase in the use of artificial intelligence and machine learning (AI/ML) methods in essentially all areas of science, technology, engineering, humanities, and education. However, despite this widespread adoption of AI/ML models as analytical tools, relatively very little is known about the theoretical properties of these models. In particular, there is minimal theory justifying their use, despite a large and growing body of empirical evidence of their efficacy.

In this talk, I will attempt to partially bridge this gap between theory and performance for a particular machine learning problem: clustering. *Clustering* is a classical machine learning task where one seeks to partition a given data into subgroups of similar items called *clusters*. I will argue that the clustering task can be modeled as a family of combinatorial optimization problems, and illustrate how to use methods from semidefinite programming to design efficient and accurate numerical methods for clustering. In particular, I will propose several models for well-behaved or *clusterable* data, where we can expect these clustering heuristics to correctly identify the hidden cluster structure. Practical applications in computer vision and astrophysics will also be discussed.

Benchmarking Electrostatic Free Energy of the Nonlinear Poisson-Boltzmann Model for the Kirkwood Sphere *Sylvia Amihere* (UA)

Abstract: Various numerical packages have been developed to solve the Poisson-Boltzmann equation (PBE) for the electrostatic analysis of solvated bio-molecules. A common benchmark test for the PBE solvers is the Kirkwood sphere, for which analytical potential and free energy are available for the linearized PBE. However, the Kirkwood sphere does not admit an analytical solution for the nonlinear PBE involving a hyperbolic sine term. In this talk, we will propose a simple numerical approach, so that the energy of the Kirkwood sphere for the nonlinear PBE can be calculated at a very high precision. Thus, providing a new benchmark test for the future developments of nonlinear PBE solvers.

Well-posedness and Stability Results for Some Nonautonomous Second-Order Evolution Equations, *Toka Diagana* (UAH)

Abstract: In this talk, we study a class of second-order evolution equations that involve time-dependent unbounded linear operators. Under some suitable assumptions, we obtain some well-posedness and stability results. Our results generalize numerous known results in the autonomous case.

Response-Aided Score-Matching Approaches for Distributed Learning under Generalized Linear Models, *Keren Li* (UAB)

Abstract: To address data localization and privacy challenges in distributed big data analysis, Li and Yang (2022) develop a representative strategy to fulfill these requirements, called Score-matching Representative (SMR) for Generalized Linear Models. Given

a data partitioning, SMR algorithm produces pseudo observation(s), named representative, on each data block by matching score function values locally. Model parameters are fitted on the collection of representatives rather than any raw data subset. We propose an efficient method called Response-Aided Score Matching Representative (RASMR) approach to improve the stability and representability of solutions to original SMR by further splitting data blocks at some model-specific response and linear predictor values. SMRs are constructed in the downstream analysis. The representatives are utilized for model parameter estimation and other statistical analysis and inference, such as link function selection, variable selection and cross-validation in big data analysis.

The Stochastic Sigmoid Beverton-Holt Equation, *Quinten McKinney* (UAH)

The sigmoid Beverton-Holt equation, given by

$$x_{n+1} = \frac{a_n x_n^\delta}{1 + x_n^\delta} \qquad x_0 > 0 \tag{1}$$

is of interest because of its applications to the population dynamics of various species. It has been studied by Harry and Kent in [1], Diagana et al in [2], Gaut et al in [3], and Bezandry et al in [4] in various cases. This presentation seeks present a proof of the existence of two distinct random almost periodic solutions to (1) under the assumption that $\delta > 1$ and a_n is a random almost periodic sequence satisfying appropriate bounds dependent upon δ.

Fourier Transforms for ODEs Without the Unique Continuation Property, *Steven Redolfi* (UAB)

Abstract: This talk is about the spectral theory associated with the differ- ential equation $Ju' + qu = wf$ on the real interval (a, b) when J is an $n \times n$ constant, invertible skew-Hermitian matrix and q and w are $n \times n$ matrices whose entries are distributions of order zero (local measures) with q Hermitian and w non-negative. Under these hypotheses it may not be possible to uniquely continue a solution from one point to another, thus blunting the standard tools of spectral theory. Despite this fact we are able to show, under certain assumptions on the coefficients of the equation, that any self adjoint restriction T of the maximal relation for the differential equation has a corresponding Fourier transform $\mathcal{F}$ and spectral space $L^2(\tau)$ into which $\mathcal{F}$ restricted to $\overline{\mathrm{dom}(T)}$ is a surjective unitary operator which diagonalizes T.