Reconstruction of Magnetic Structures from Spacecraft Data: A Test Study for Voyagers in the Heliosheath

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Abstract

This study presents a C++ code implementing the Grad-Shafranov equation for reconstructing two-dimensional coherent magnetic structures from single-spacecraft data. The code follows the approach of Hau and Sonnerup [1999], where they recovered magnetic field maps of magnetopause current layer crossings using AMPTE/IRM measurements. We validated our initial code by reproducing their benchmark results, accurately reconstructing the magnetic islands and X-line structures embedded within the magnetopause layer. Using real AMPTE magnetopause crossings data, we graphed the pressure, vector potential, and different axes of the magnetic field against each other to recreate the magnetopause crossings studied by Hau and Sonnerup. Numerical methods preprocess the data to provide the proper frame velocity, constraints, and higher resolution spatial increments along the spacecraft trajectory. The Grad-Shafranov equation demonstrates the mapping of plasma equilibrium and coherence structures by generating 2D magnetic field representations from single-spacecraft observations. Further application to Voyager 1's sparse heliosheath measurements has the potential to provide insights and enhance the restricted direct observations from this distant frontier region.

Introduction

The heliosheath forms the outer solar wind boundary, stretching from the termination shock to the heliopause. Voyager 1 uniquely traversed this sparsely sampled frontier, returning limited magnetic field data. The Grad-Shafranov equation can analyze heliosheath coherent structures, enhancing the value of these rare observations and plasma equilibrium insights.

This study develops a code implementing the Grad-Shafranov technique which describes magnetohydrodynamic (MHD) equilibrium in 2D structures with an invariant axis. The equation can be solved to reconstruct magnetic island features formed by reconnection. Following Hau and Sonnerup, the code recovers 2D field maps from individual spacecraft crossings. Applied to Voyager 1 heliosheath data, it can characterize plasma equilibrium as Voyager 1 entered interstellar space.

Method

The Grad-Shafranov code was developed in C++ with the use of one external library. The equation assumes static MHD equilibrium with a magnetic field confined to the xy plane, making the z component constant reducing the problem to 2D. With spacecraft measurements as initial values, we integrated the Grad-Shafranov equation (1) in small steps perpendicular to the trajectory of the simulation spacecraft to map the area.

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = -4\pi \frac{dP_t}{dA} \qquad (1)$$

The total pressure P_t (plasma + magnetic) (2) is represented as a function of the vector potential A which is a function of x and y magnetic field components, A(x,y).

$$P_t = p + \frac{B^2}{8\pi} \qquad (2)$$

A 2nd-order Taylor expansion calculates A(x,y) at each new point (3). We sampled A and P_t at regular intervals along the trajectory. For the benchmark case, a 2D mesh was addressed through x and y indices. Taylor expansion of A in y allowed extrapolating A away from the trajectory (4).

$$A(x, y+\Delta y) \simeq A(x,y) + \left(\frac{\partial A}{\partial y}\right)_{x,y} \Delta y + \frac{1}{2}\left(\frac{\partial^2 A}{\partial y^2}\right)_{x,y} (\Delta y)^2 \qquad (3)$$

$$A_{i,j+1} \simeq A_{ij} + \frac{\partial A}{\partial y}\Big|_{ij} \Delta y + \frac{\partial^2 A}{\partial y^2}\Big|_{ij} \frac{\Delta y^2}{2} = A_{ij} + B_{x,ij}\,\Delta y + \frac{\partial^2 A}{\partial y^2}\Big|_{ij} \frac{\Delta y^2}{2} \qquad (4)$$

We calculate the second x-derivative of the vector potential A directly using finite differences. This allows initializing and extrapolating the x-component of the magnetic field, $B_{x,i,j+1}$ using the Taylor expansion. We increment the index j and repeat this process until the full 2D mesh is completed. When applying the code to real AMPTE spacecraft data, other key components are implemented. These include: interpolating the fields onto a regular spatial grid using cubic splines; computing the De Hoffmann-Teller reference frame velocity by least-squares fitting; rotating the coordinate system to find the optimal invariant z-axis orientation. Additionally, we calculate the mean magnetic field by averaging over all samples. A smooth differentiable curve is also fitted to the measured total pressure Pt versus vector potential A. This fitted Pt(A) function generates the source term for the Grad-Shafranov equation.

Results

The following graphs are accurate reconstructed results of Hau and Sonnerup's Figure 6 (reconstruction of AMPTE/IRM event), measured in CGS units.

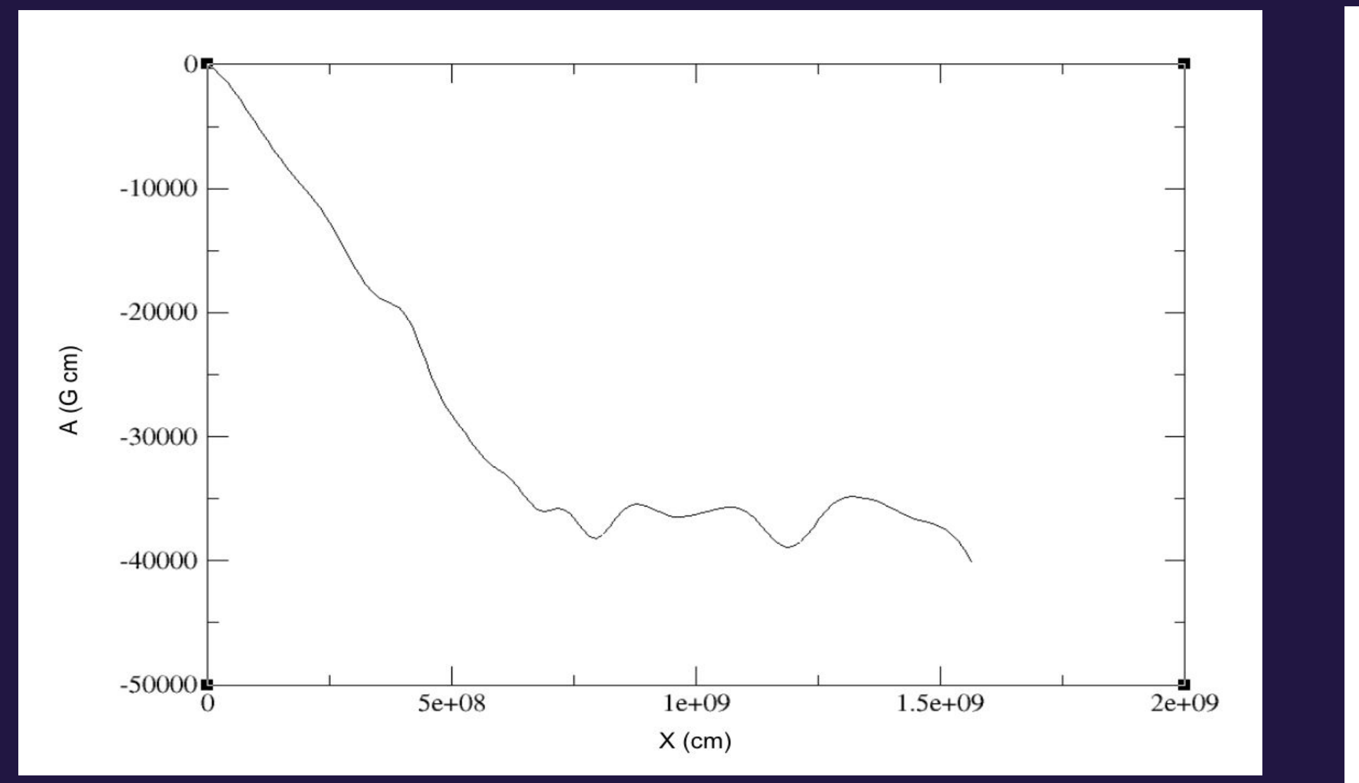

Figure 1: Shown in this graph is the magnetic vector potential A along the X-axis spacecraft trajectory.

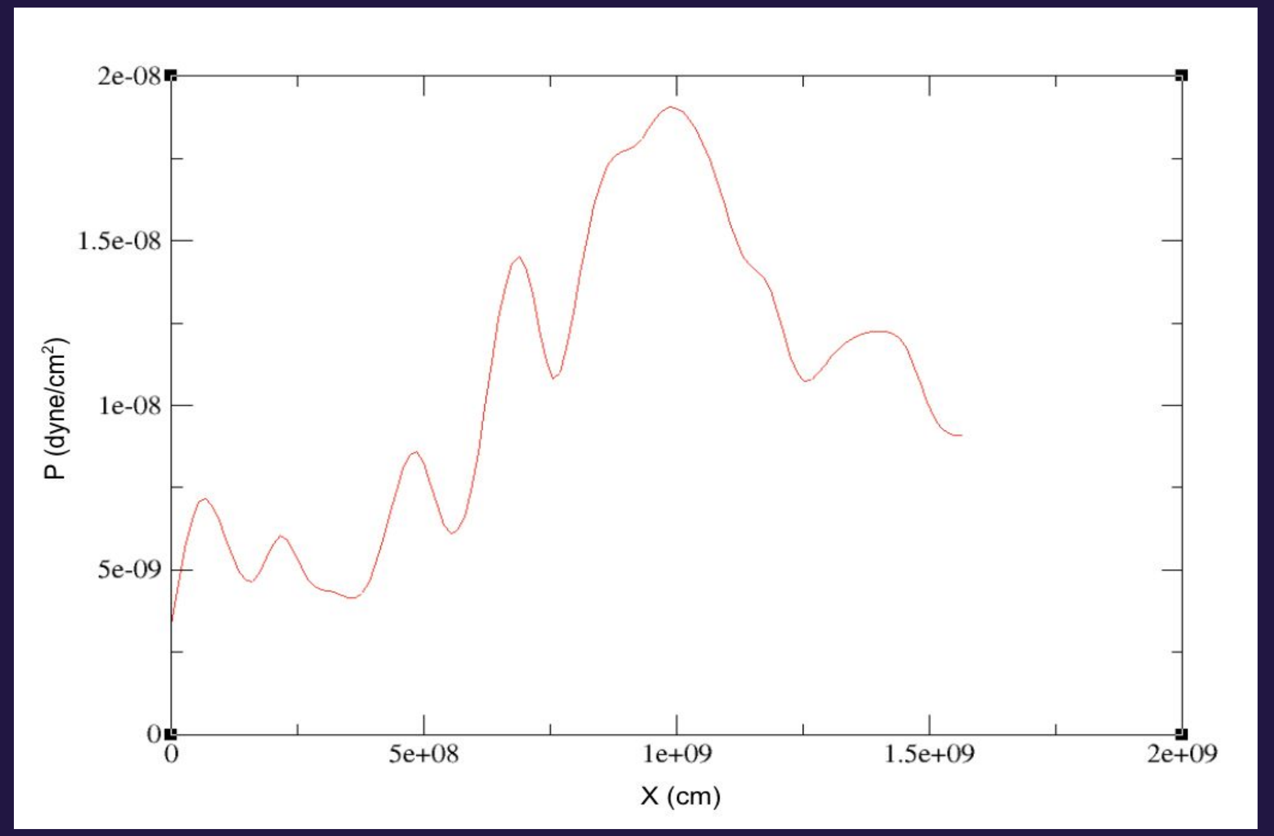

Figure 2: This graph plots the plasma pressure P along the X-axis spacecraft trajectory.

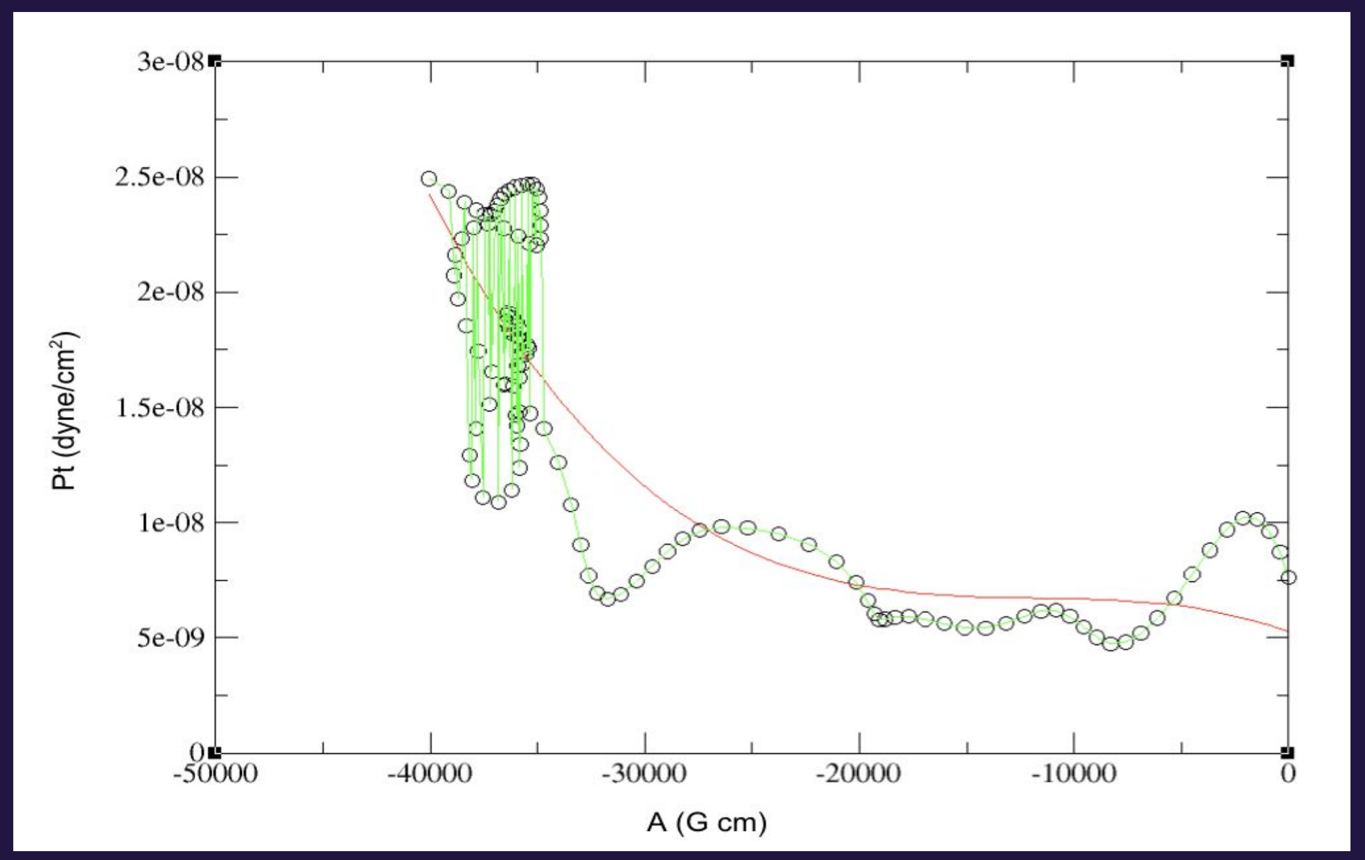

Figure 3: This graph shows the total pressure P_t as a function of A. The green line connects the actual data points, which have a zigzag pattern indicating the pressure value is alternating up and down even though the points are close together in A. The black circles are interpolated points added through cubic spline interpolation. They closely follow the trend of the real data. The red line shows a monotonic decreasing fit to the data, even though the actual data points start increasing again at a higher A value illustrating the complexities of real magnetopause structure.

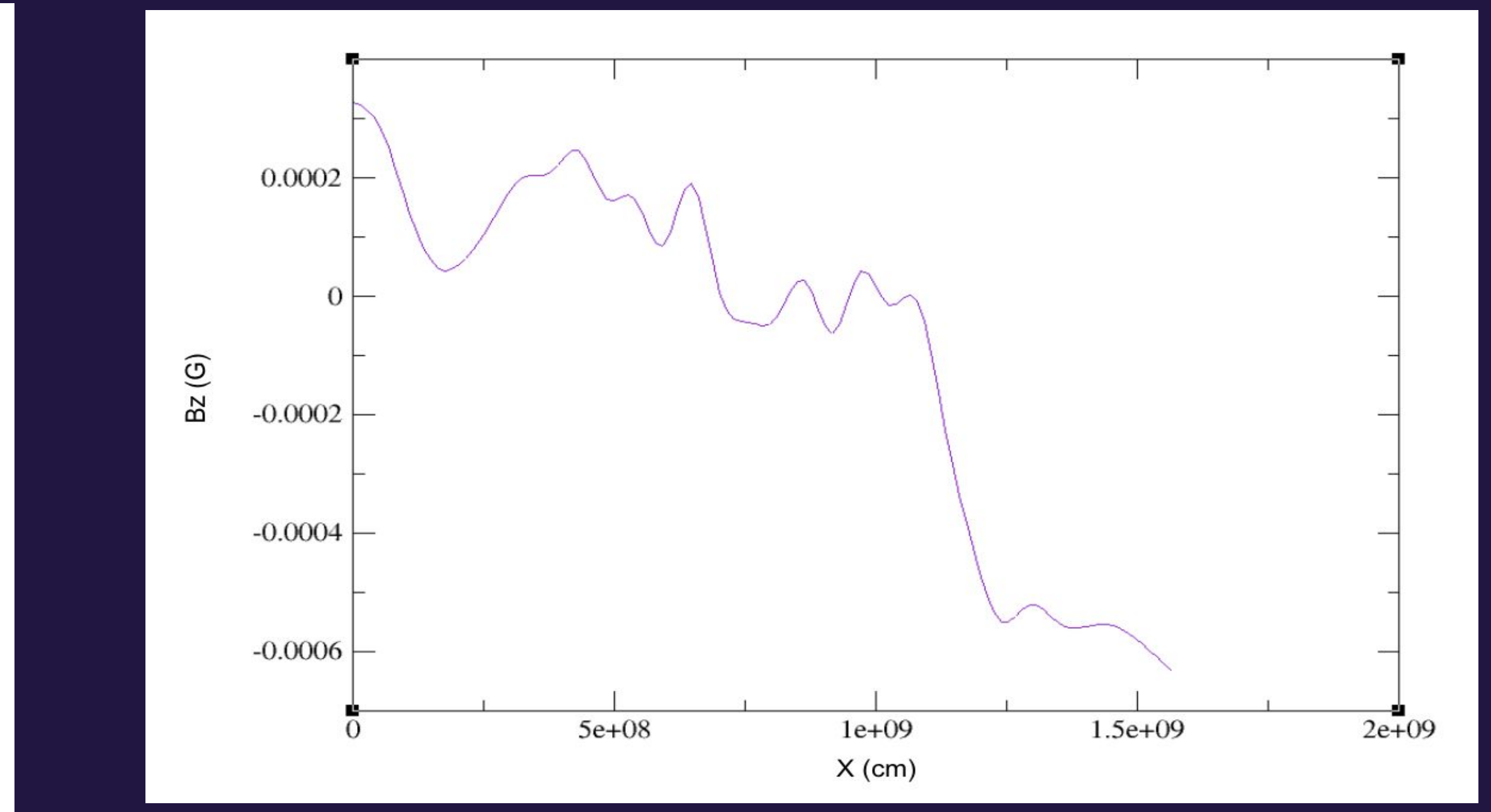

Figure 4: This graph plots the Bz component along the X-axis spacecraft trajectory

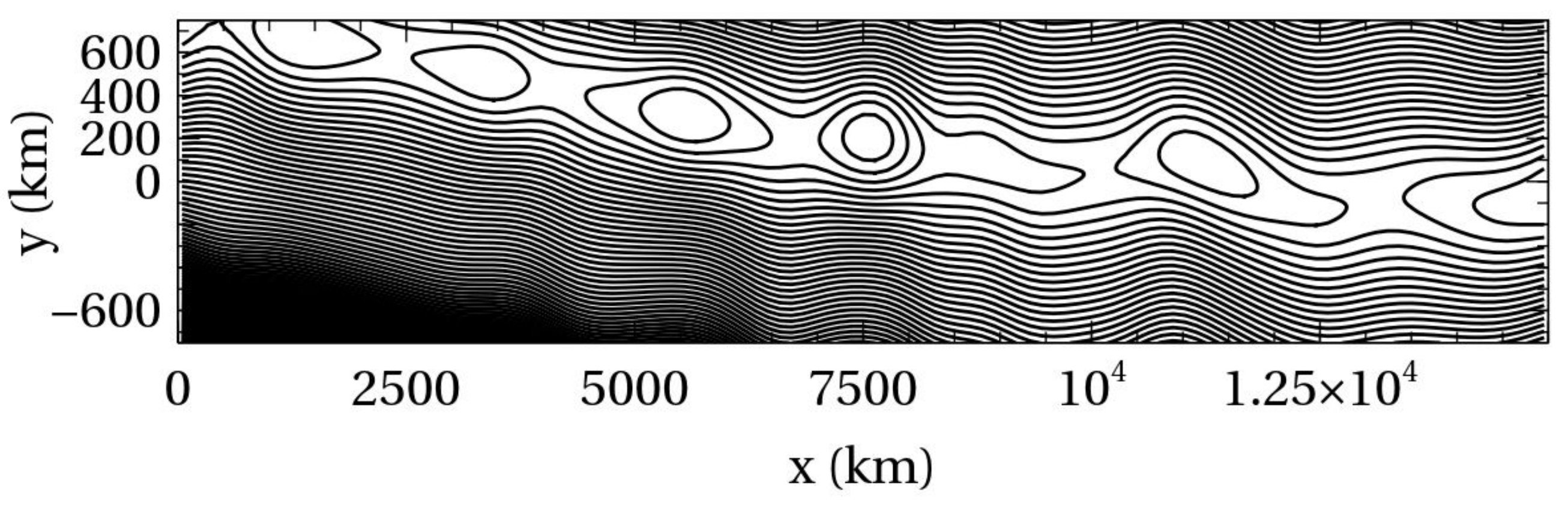

Figure 5: This contour plot provides a 2D visualization of the coherent magnetic island structures embedded within the larger magnetopause current layer, recovered from AMPTE spacecraft observations. The X axis is the spacecraft trajectory through the magnetopause and the Y axis represents the direction perpendicular to both the magnetopause normal and the spacecraft trajectory.

Conclusions and Future Work

While we faced many challenges and errors along the way, in the end we successfully reproduced the results of Hau and Sonnerup's seminal magnetopause analysis, starting completely from scratch. This process was tedious, requiring accurately implementing many line fits, derivatives, and numerical integration schemes to achieve the proper results. There were times when the solution was unclear from the paper alone, so we had to utilize outside resources to fully grasp the method and determine the right approach for the current step. Our reproduction of the results is a huge accomplishment for our two-person team.

With the finalized code, we can apply it to reconstruct Voyager 1 data across the heliopause. However, since Voyager 1's plasma instrument stopped working in 1980, modifications will be needed such as identifying a suitable proxy for the plasma pressure.

References

Hau, L.-N., and Bengt U. Sonnerup. "Two-Dimensional Coherent Structures in the Magnetopause: Recovery of Static Equilibria from Single-Spacecraft Data." Journal of Geophysical Research: Space Physics, vol. 104, no. A4, 1999, pp. 6899–6917, https://doi.org/10.1029/1999ja900002.

Acknowledgements

This work was supported by funding from NSF grant AGS-1950831 for the UAH CSPAR/NASA MSFC Heliophysics REU program and NASA 18-2HSWO218_2-0003.

I would like to thank Dr. Florinski for all his guidance and patience throughout the summer and for inviting me to coffee chats - I found them very refreshing. Thank you to the 2023 UAH REU team and all the interns for being so kind and supportive.